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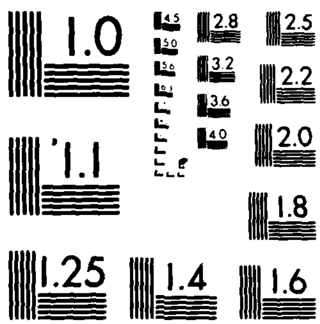

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THE OPTIMISATION OF THE ARRAY CONFIGURATION FOR A SPLIT BEAM SONAR SYSTEM (U)

BY

R.J. WYBER

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THE OPTIMISATION OF THE ARRAY CONFIGURATION FOR A SPLIT BEAM SONAR SYSTEM (U)

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ABSTRACT

In a split beam sonar system the signal to noise ratio and the azimuthal resolution may be altered by varying the configuration of the subarrays. This paper analyses the performance of such a sonar, taking account of the noise correlation between the subarrays to optimise the sonar performance. (U)

POSTAL ADDRESS: The Superintendent, RAN Research Laboratory, P.O. Box 706, Darlinghurst. N.S.W. 2010.

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THE OPTIMISATION OF THE ARRAY CONFIGURATION FOR A SPLIT BEAM SONAR SYSTEM

1. INTRODUCTION

A common requirement in the design of a sonar is to achieve a high azimuthal resolucion subject to constraints on the sonar array size. One method of doing this is to use a split beam technique in which the array is split into two subarrays which both ensonify the sonar target. The target resolution is then determined by measuring the time difference between the signal returns for the two arrays. The measurement of the time difference is usually carried out by measuring the phase difference between the signals received at each subarray.

For a given overall array size the performance of such a sonar may be varied by altering the dimensions of the two subarrays. When this is done the noise measured at each subarray may not generally be assumed independent. This paper analyses the performance of a split beam sonar under these conditions to determine the optimum configuration of the subarrays.

2. THE AMPLITUDE AND PHASE AT THE SUBARRAYS

Suppose that a split beam sonar system consists of two sub-arrays of width b and a spacing between array centres of d. The signal received at the arrays is resolved to a number of range cells. Then for each range cell the amplitude of the received signal is calculated by adding the amplitude received at each sub-array, and the phase is calculated by subtracting the phase received at each subarray. It will be assumed that the phase calculated for each subarray is referenced to the reflected target signal received at the electrical centre of the total receiving array. This point is marked 0 in Fig. 1. It is assumed that in any range cell the signal reflected from the target may be represented by a reflection from one azimuthal point. If this point subtends an angle β to the array axis then from Fig. 1 the electrical signal received at subarray one is advanced by a phase θ where

$$\theta = (kd \sin \beta)/2 \qquad (1)$$

where k is the wave number for the transmitted frequency.

The signal received at subarray two is retarded by θ. This is illustrated in Fig. 2. Thus in the absence of noise the measured phase difference θ_d is given by

$$\begin{aligned} \theta_d &= 2\theta \\ &= kd \sin \beta \end{aligned} \qquad (2)$$

As the signal received from the array is contaminated by noise the measured signals received at the array are given by

$$\tilde{m}_1 = \tilde{S}_1 + \tilde{n}_1$$

and $$\tilde{m}_2 = \tilde{S}_2 + \tilde{n}_2$$

where $\tilde{m}_1$ and $\tilde{m}_2$ have phase angles ϕ_1 and ϕ_2 and $\tilde{n}_1$ and $\tilde{n}_2$ are the effective noise vectors at each array. The angle between $\tilde{n}_1$ and $\tilde{S}_1$ is assumed to be ψ_1 and $\tilde{n}_2$ and $\tilde{S}_2$ to be ψ_2.

Now

$$\tan (\phi_i - \theta) = n_i \sin \psi_i/(S_i + n_i \cos \psi_i) \qquad (3)$$

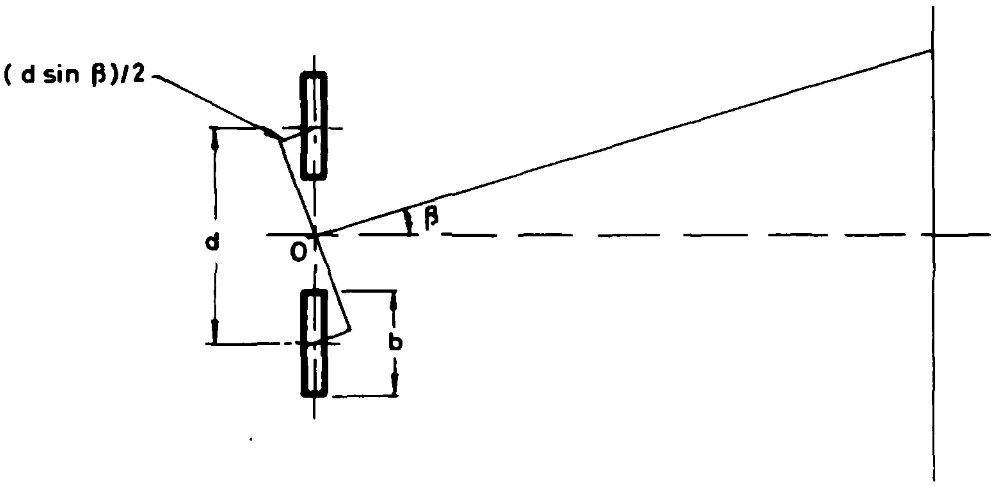

Fig. 1. The geometry of the array.

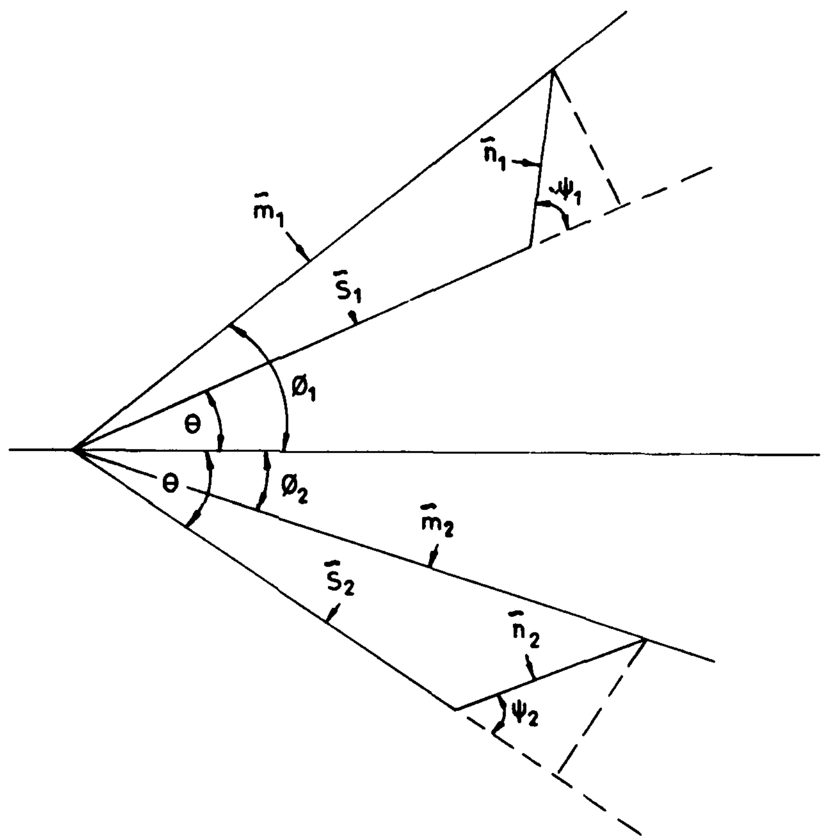

Fig. 2. The relationship of $\bar{m}$ to $\bar{n}$ and $\bar{s}$.

and $m_i^2 = (S_i + n_i \cos\psi_i)^2 + (n_i \sin\psi_i)^2$ (4)

for $i = 1, 2,$ $S_i = |\tilde{S}_i|$ and $n_i = |\tilde{n}_i|$

In n is not too large (3) reduces to

$$\Delta\theta_i = \emptyset_i - \theta$$

$$= n_i \sin(\psi_i)/S \quad \text{where} \quad S = S_1 = S_2$$

Then if n_i is assumed to be Gaussian ψ_i uniformly distributed and E $(\cdot)$ denotes the expected value

$$E(\Delta\theta_i)^2 = (n_i{}^2/S^2)(1/2\pi)\int_0^{2\pi} \sin^2\psi_i \, d\psi_i$$

$$= (n_i{}^2/2S^2) \tag{5}$$

$$= (1/2\,S_e)$$

where S_e is the expected signal power divided by the expected noise power at the subarray. Also

$$E(m_i{}^2) = S^2 + n_i{}^2 (\cos^2\psi_i + \sin^2\psi_i)$$

$$+ 2\,Sn_i(1/2\pi)\int_0^{2\pi} \cos\psi_i \, d\psi_i$$

$$= S^2 + n_i{}^2 \tag{6}$$

As $E(\Delta\theta_i) = 0$

and $E(m_i) = S$

the standard deviations of $\Delta\theta$ and m are given by

$$Sd(\Delta\theta_i) = 1/(2S_e)^{1/2}$$

and $Sd(m_i) = n_i$

3. THE EFFECT OF INDEPENDENT INTERFERING NOISE

Now if $\tilde{n}_1$ and $\tilde{n}_2$ may be assumed independent then as the measured phase error is given by

$$\Delta\theta = \Delta\theta_1 - \Delta\theta_2$$

the standard deviation of the measured phase error is given by

$$Sd(\Delta\theta) = 1/(S_e)^{\frac{1}{2}} \tag{7}$$

A more useful form of expressing (7) is to let S_x be the signal to noise ratio required to give a standard deviation of $Sd(\Delta\theta)$.

Then

$$S_x = - 20 \log (Sd(\Delta\theta)) \tag{8}$$

Also the standard deviation in the amplitude found by summing the two sub-arrays is given by

$$Sd(m_1 + m_2) = \sqrt{2}\, n$$

As $$E(m_1 + m_2) = 2\, S$$

The signal to noise ratio for the amplitude is given by

$$S_a = 4S^2/ Sd^2(m_1 + m_2)$$

$$= 2\, S_e \tag{9}$$

This represents 3 dB improvement in the signal to noise ratio relative to the signal to noise ratio at the subarray.

4. THE EFFECT OF CORRELATED INTERFERING NOISE ON THE PHASE AND AMPLITUDE

Now if the interfering noise is due to sea noise or reverberation it may no longer be valid to consider $\tilde{n}_1$ and $\tilde{n}_2$ to be independent. To analyse this situation it is necessary to consider the distributed nature of the noise. As illustrated in Fig. 3 suppose that the noise power at angle β in an element of width $\Delta\beta$ is given by $A^2(\beta)\Delta\beta$. The noise field may then be represented by a number of sources of strength

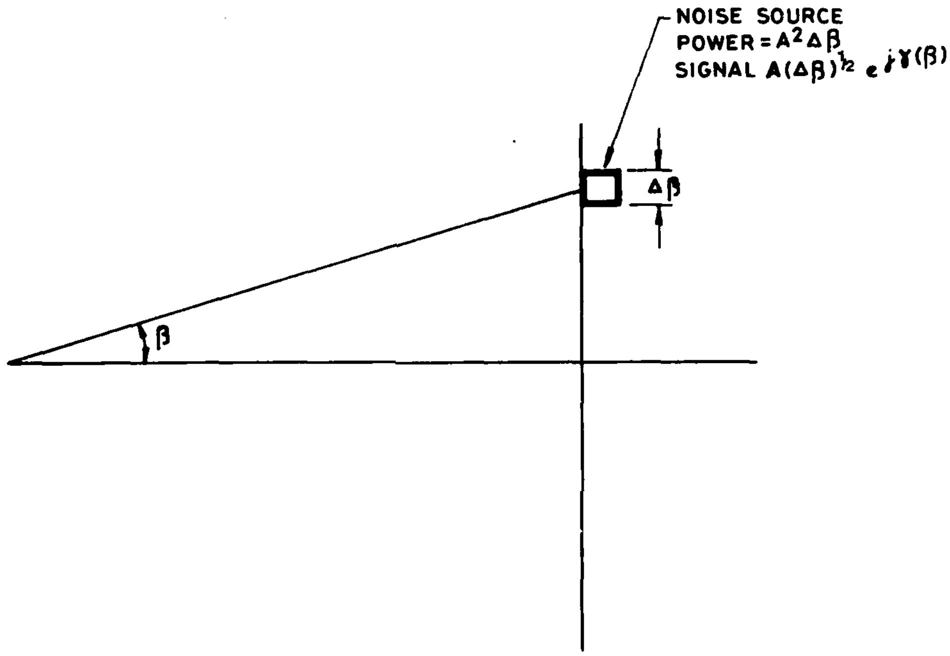

Fig. 3. The noise distribution.

$$p_h \quad = \quad A_h \exp(j\gamma_h)\Delta\beta^{\frac{1}{2}} \quad \text{for } h = -\pi/2\Delta\beta \text{ to } \pi/2\Delta\beta \qquad (10)$$

where $A_h \quad = \quad A(h\Delta\beta)$

$$\gamma_h \quad = \quad \gamma(h\Delta\beta)$$

and $\gamma(\beta)$ is the phase of the noise generated at an angle β. Then if $R(\beta)$ is the subarray response to a noise source at β the noise received from the h th element at subarray one has an amplitude

$$n_{1_h} \quad = \quad A_h\ R_h\ (\Delta\beta)^{\frac{1}{2}} \qquad (11)$$

where $R_h \quad = \quad R(h\Delta\beta)$

and phase d_{1_h} given by

$$d_{1_h} \quad = \quad \gamma_h + kd\ \{\sin\ (h\Delta\beta)\}\ /2 \qquad (12)$$

Similarly at subarray two

$$r_{2_h} \quad = \quad A_h\ R_h\ (\Delta\beta)^{\frac{1}{2}}$$

and $d_{2_h} \quad = \quad \gamma_h - kd\{\sin\ (h\Delta\beta)\}\ /2 \qquad (13)$

Then if,

$$\psi_{1_h} \quad = \quad d_{1_h} - \theta$$

and $\psi_{2_h} \quad = \quad d_{2_h} + \theta$

the component in quadrature to $\tilde{S}_i$ is given by

$$n_{ih} \sin \psi_{ih} \quad \text{for } i = 1,\ 2$$

and the component in the direction of S_i is given by

$$n_{ih} \quad \cos\ \psi_{ih} \text{ for } h = 1,\ 2$$

Thus if $Sd(\Delta\theta_h)$ is the standard deviation of the phase error due to

the hth noise source

$$Sd(\Delta\theta_h)^2 = E\ \{(A_h{}^2/S^2)R_h{}^2\}\ \{\sin\ (\gamma_h + kd(\sin\ \beta_h)/2 - \theta)$$

$$- \sin\ (\gamma_h - kd\ (\sin\ \beta_h)/2 + \theta)\}^2\ \Delta\beta$$

where $\beta_h = h\Delta\beta$

i.e.

$$Sd(\Delta\theta_h)^2 = E\ \{\ (A_h{}^2\ R_h{}^2)/S^2\}\ \{4\ \cos^2\ (\gamma_h)\ \sin^2\{k_d(\sin\ \beta_h)/2 - \theta\}\ \Delta\beta \quad (15)$$

For any β_h, γ_h may be assumed to be uniformly distributed as each range cell covers a number of wavelengths at the frequency of operation.

Then as $E\ \{\cos^2(\gamma_h)\} = 1/2$

$$Sd(\Delta\theta_h)^2 = 2(A_h{}^2/S^2)\ R_h{}^2\ \sin^2\ \{\ kd(\sin\ \beta_h)/2 - \theta\}\Delta\beta \quad (16)$$

Similarly

$$Sd(m_{ih})^2 = 2A_h{}^2\ R_h{}^2\ \cos^2\ \{kd(\sin\ \beta_h)/2 - \theta\}\ \Delta\beta \quad (17)$$

The square of the standard deviation of the sum of the components Δm_{ih} and $\Delta\theta_{ih}$ is found by summing the square of the standard deviation of each component.

Thus

$$Sd(\Delta\theta)^2 = \sum\ Sd\ (\Delta\theta_h)^2$$

The summation may be replaced by integration as $\Delta\beta$ is made arbitrarily small so that

$$Sd(\Delta\beta)^2 = (2/S^2)\int_{-\pi/2}^{\pi/2} A^2(\beta)R^2(\beta)\ \sin^2\ \{kd\ \sin\ \beta/2 - \theta\}\ d\beta$$

It is now assumed that the noise is isotropic so that $A^2(\beta) = A^2$

and is independent of β and the array is assumed unshaded and continuous so that

$$R(\beta) = \sin((kb \sin\beta)/2)/((kb \sin\beta)/2) \qquad (18)$$

For $kb >> 1$ $\sin\beta \simeq \beta$ except where $R(\beta) \simeq 0$.

Then letting $u = k\,d/2$ and $\nu = k\,b/2$

$$Sd(\Delta\theta)^2 = 2(A/S)^2 \int_{-\pi/2}^{\pi/2} \sin^2(\nu\beta)\,\sin^2(u\beta - \theta)/(\nu\beta)^2\,d\beta \qquad (19)$$

$$= 2(A/S)^2 \int_{-\pi/2}^{\pi/2} \{\sin^2(\nu\beta)\,\sin^2(u\beta)\,\cos^2\theta + \sin^2(\nu\beta)$$

$$\cos^2(u\beta).\ \sin^2\theta\,\}/(\nu\beta)^2\,d\beta$$

From [1]

$$\int_{-\pi/2}^{\pi/2} \sin^2(\nu\beta)\,\sin^2(u\beta)/\beta^2\,d\beta = (\pi/2)\ \{\min(u,\nu)\}$$

and

$$\int_{-\pi/2}^{\pi/2} \sin^2(\nu\beta)\,\cos^2(u\beta)/\beta^2\,d\beta = (\pi/2)\ \{2\nu - \min(u,\nu)\} \qquad (20)$$

Thus

$$Sd(\Delta\theta)^2 = (A^2/S^2)\,\pi\,\{\min(u,\nu)\cos^2\theta + (2\nu - \min(u,\nu))\sin^2\theta\}/\nu^2 \qquad (21)$$

Also the total noise received by the arrays is given by

$$n^2 = A^2 \int_{-\pi/2}^{\pi/2} \sin^2(\nu\beta)/(\nu\beta)^2\,d\beta = A^2\pi/\nu \qquad (22)$$

Therefore,

$$Sd(\Delta\beta)^2 = n^2/S^2\ \{\cos^2\theta + \sin^2\theta\}$$

$$= n^2/S^2$$

$$= 1/S_e \qquad \text{for } \nu \leq u$$

and

$$Sd(\Delta\theta)^2 = 1/S_e \{(u/\nu) \cos^2 \theta + (2 - u/\nu)\sin^2 \theta\} \qquad (23)$$

for $\nu > u$

Similarly

$$Sd(m)^2 = n^2 \qquad \text{for } \nu \leq u$$

$$= n^2 \{(2 - u/\nu) \cos^2 \theta + (u/\nu) \sin^2 \theta\} \qquad (24)$$

for $\quad u$

The phase error is minimised if $\theta = 0$ which may be achieved at long ranges by centering the target in the beam.

Then

$$Sd(\Delta\theta)^2 = (d/b\ S_e) \qquad \text{for } b > d$$

$$= (1/S_e) \qquad \text{for } b \leq d$$

i.e. $$Sd(\Delta\theta)^2 = \min(d,b)/b\ S_e \qquad (25)$$

Similarly as the signal to noise ratio is given by

$$S_a = 4\ S^2/(2\ Sd(m)^2)$$

$$S_a = 2\ S_e \qquad \text{if } d \geq b$$

$$= 2\ S_e/\{2 - b/d\} \qquad \text{if } d < b$$

i.e. $$S_a = 2\ S_e b/(2\ b - \min(b,d)) \qquad (26)$$

5. THE AZIMUTHAL ERROR

To determine the standard deviation of the azimuthal error from the standard deviation of the phase error it may be noted in Fig. 1 that if a reflecting point at a range r is a distance h off the array axis then

$$h = r \sin \beta \qquad (27)$$

Then from (2)

$$(h/r) = \theta_d/kd$$

and thus $S_d(h) = (r/kd)\ S_d(\Delta\theta)$

Then substituting the value for $S_d(\Delta\theta)$ from (25)

$$S_d(h)^2 = (r^2/k^2d^2)(\min\ (b,d)/b\ S_e) \qquad (28)$$

From (22) $n^2 = A^2\pi/V$

$$= 2\pi A^2/kb$$

Then $S_e = S^2/n^2$

$$= S^2kb/2\pi A^2 \qquad (29)$$

Substituting (29) in (28) gives

$$S_d(h)^2 = ((r^2/k^2)(2\pi A^2/kS^2)(\min\ b,d))/d^2b^2$$

$$= q\ \min\ (b,d)/d^2b^2 \qquad (30)$$

where $q = r^2A^2 2\pi/k^3S^2$

Similarly from (26)

$$S_a = 2\ S_e\ b/(2b - \min(b,d))$$

$$= (S^2k/\pi A^2)(b^2/(2\ b - \min(b,d)))$$

$$= p\ b^2/(2\ b - \min(b,d))$$

where $p = (S^2\ k/\ \pi\ A^2) \qquad (31)$

6. THE OPTIMUM ARRAY CONFIGURATION

Now if the total width of the array is c the distance between the subarray centres, d may be varied by altering the subarray width b as shown in Fig. 4.

Then as $d = c - b$

$$Sd(h)^2 = q\ (\min(c - b),b)/b^2(c - b)^2$$

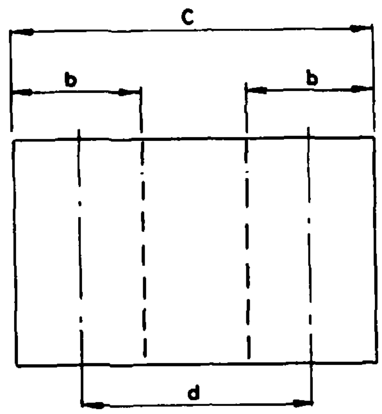

Fig. 4. The relationship of d to b and c.

If $b < c/2$ then $Sd(h)$ is a minimum when

$$\partial/\partial b\ (1/b(c - b)^2) = 0$$

or $$b = c/3 \tag{32}$$

If $b \geqslant c/2$ then $Sd(h)^2$ is a minimum when

$$\partial/\partial b\ (1/b^2(c - b)) = 0$$

or $$b = 2c/3 \tag{33}$$

At both these minima

$$Sd(h)^2 = 6.75\ q/c^3 \tag{34}$$

From (31) if $$b = 2c/3 \tag{35}$$

$$S_a = 4\ p\ c/9$$

If $$b = c/3$$

$$S_a = p\ c/3 \tag{36}$$

Thus of the two solutions (32) and (33) which maximise the azimuthal resolution setting $b = 2c/3$ gives a signal to noise ratio which is 1.2 dB greater than setting $b = c/3$

Alternatively if optimisation is carried out for the signal to noise ratio of the amplitude, then as

$$S_a = p\ b^2/(2b - \min\ (b,d))$$

For $b < d$

$$S_a = p\ b^2/(2b - b)$$

Then $\partial S_a/\partial b \quad = \quad p$ (37)

Thus the signal to noise ratio increases with b for $b < c/2$

If $\quad b > d$

$$S_a = p\, b^2/(2b - (c - b))$$

$$= p\, b^2/(3b - c)$$

Then $\quad \partial S_a/\partial b$

$$= p\,\{ (2b(3b - c) - b^2 3)/(3b - c)^2 \}$$

$$= p\,\{ (3b^2 - 2\,bc)/(3b - c)^2 \}$$

$$= 0 \quad \text{when} \quad b = 2c/3 \qquad (38)$$

This is the only turning point in the region $c/2 < b < c$ and corresponds to a minimum. Thus S_a is a maximum in this interval if

$$b = c/2$$

$$\text{or} \quad b = c \qquad (39)$$

When $\quad b = c/2$

$$S_a = p\, c/2$$

When $b = c \quad S_a = p\, c/2$ (40)

Thus a dual optimum exists for S_a . For $b = c$ however the azimuthal error tends to infinity. Thus if it is required to optimise the signal to noise ratio of the array b should be set equal to $c/2$.

Then $\quad S_d\,(h)^2 \quad = \quad 8\,q/c^3$ (41)

Relative to the solution of (34) this gives a 0.5 dB improvement in the signal to noise ratio for the amplitude, at the expense of a degradation in the phase performance which corresponds to a signal to noise loss of 0.7 dB. The variations in S_a and $S_d(h)$ as b is changed are illustrated in Fig. 5.

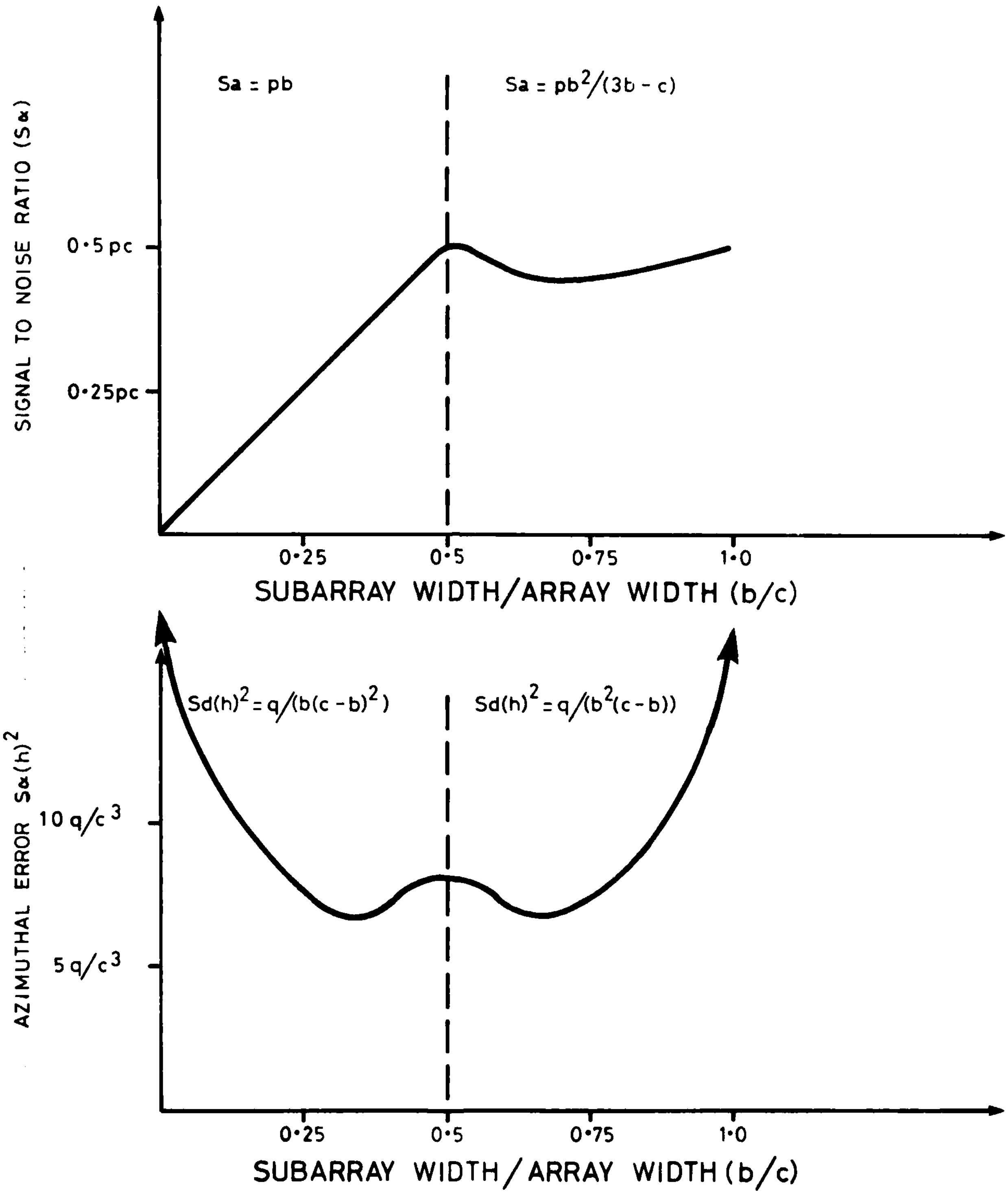

Fig. 5. The variation of the phase error and the signal to noise ratio with Subarray Width.

7. CONCLUSION

A split beam sonar using unshaded subarrays has been analysed. For such a sonar the maximum azimuthal resolution is obtained when the two subarrays are each two thirds of the width of the full array width and the maximum signal to noise ratio is obtained when the subarrays are one half of the full array width.

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16. Abstract

In a split beam sonar system the signal to noise ratio and the azimuthal resolution may be altered by varying the configuration of the subarrays. This paper analyses the performance of such a sonar, taking account of the noise correlation to optimise the sonar performance.